ISSN 2229-5518

Relation between N numbers and their LCMs and GCDs

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Abstract— This paper explains the relation among product of 'n' natural numbers and the LCMs and GCDs of all combinations of these numbers. It is known that the product of two numbers is equal to the product of their LCM and GCD. There is also a relation for three numbers. In this paper I am going to introduce two generalized relations for 'n' natural numbers.

Index Terms — N numbers , Product , LCM (Least Common Multiple) , GCD (Greatest Common Multiple) , Relation-1, Relation-2 , Odd , Even

1 Introduction

We know that if a, b are two numbers then their product is equal to product of their LCM and GCD

i.e. axb =LCM (a,b) × GCD (a,b)

and also we know that if a,b,c are 3 numbers then

LCM (a,b,c) = [a × b × c × GCD (a,b,c)] /[GCD (a,b) × GCD (b,c) × GCD(c,a)]

GCD (a,b,c) = [a × b × c × LCM (a,b,c)] /[LCM (a,b) × LCM (b,c) × LCM(c,a)]

In this paper I am going to introduce two generalized formulae and their proof for 'n' natural numbers. The product of 'n' numbers can be expressed in terms of LCMs and GCDs of different combinations in two different ways as Relation 1 and Relation 2.

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2 Relations

The Product of 'n' numbers can be expressed in two forms i.e, Relation 1 and Relation 2 as below:

Let $a_1,a_2,a_3……….a_{n-1}, a_n$ be 'n' natural numbers

Relation - 1

$$a_1 \times a_2 \times a_3 \dots\dots\dots a_{n-1} \times a_n = \frac{\text{The product of GCDs of all sets containing even no. of Elements} \times \text{LCM(of all numbers.)}}{\text{The product of GCDs of all sets containing odd no. of elements (except 1)}}$$

Relation - 2

$$a_1 \times a_2 \times a_3 \dots\dots\dots a_{n-1} \times a_n = \frac{\text{The product of LCMs of all sets containing even no. of Elements} \times \text{GCD(of all numbers.)}}{\text{The product of LCMs of all sets containing odd no. of elements (except 1)}}$$

3 Proof of Relation 1

1st Relation

ISSN 2229-5518

Now let us prove the 1st Relation i.e.

$$a_1 \times a_2 \times a_3 \ldots\ldots\ldots a_{n-1} \times a_n = \frac{\text{The product of GCDs of all sets containing even no. of Elements} \times \text{LCM(of all numbers.)}}{\text{The product of GCDs of all sets containing odd no. of elements (except 1)}}$$

Let $a_1, a_2, a_3 \ldots\ldots\ldots a_{n-1}, a_n$ be 'n' natural numbers

The above equation can be rearranged as

$$1 = \frac{a_1 \times a_2 \times a_3 \ldots\ldots\ldots a_{n-1} \times a_n \times \text{The product of GCDs of all sets containing odd no. of elements(except 1)}}{\text{The product of GCDs of all sets containing even no. of Elements} \times \text{LCM(of all numbers)}}$$

Let

$a_1 = 2^{(x_1)} \times 3^{(y_2)} \times 5^{(z_3)} \ldots\ldots\ldots\ldots\ldots$

$a_2 = 2^{(x_3)} \times 3^{(y_4)} \times 5^{(z_1)} \ldots\ldots\ldots\ldots\ldots$

:

:

:

$a_n = 2^{(x_n)} \times 3(y_1) \times 5^{(z_{n-1})} \times \ldots\ldots\ldots\ldots\ldots.$

(all numbers are taken in random)

(all exponents are whole numbers only)

Let us assume

$x_1 \le x_2 \le x_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots x_{n-1} \le x_n$

$y_1 \le y_2 \le y_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots y_{n-1} \le y_n$

and so on.

Similarly with all other exponents of prime factors.

Let us take <u>Numerator</u>

$a_1 \times a_2 \times a_3 \ldots\ldots\ldots a_{n-1} \times a_n$ × The product of GCDs of all sets

containing odd no. of elements(except 1) = $2^q \times 3^r \times 5^s \ldots . p^t$

For some $\{q, r, s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots t\} \in N$

LCM (or GCD) of n numbers = Product of LCM (or GCD) of each of its prime factors with same base.

So for 2 powers we apply the above relation

We take elements as A= $\{2^{(x_1)}, 2^{(x_2)} \ldots\ldots . 2^{(x_{n-1})}, 2^{(x_n)}\}$

$(x_1 \le x_2 \le x_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots x_{n-1} \le x_n)$

Let solution is $=2^q$

To know the value of q it is necessary to find the number of sets

No. of sets containing odd no. of elements with $2^{(x_1)}$ as GCD are :

With 3no`s $\{2^{(x_1)}, a, b\}$ a and b are any two numbers of set A. There can be ${}^{n-1}C_2$ such sets possible.

With 5no`s $\{2^{(x_1)}, a, b, c, d\}$ a, b, c and d are any 4 numbers of set A. There can be ${}^{n-1}C_4$ such sets possible

………… and so on.

No of sets containing odd no. of elements with $2^{(x_2)}$ as GCD are :

With 3no`s $\{2^{(x_2)}, a, b\}$ a and b are any two numbers of set A. There can be ${}^{n-2}C_2$ such sets possible

With 5no`s $\{2^{(x_2)}, a, b, c, d\}$ a, b, c and d are any 4 numbers set A. There can be ${}^{n-2}C_4$ such sets possible

………………..and so on

Solution $=2^q$

If we multiply GCD s (odd) and product of the numbers, we get

$q = x_1\{{}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\} + x_2\{{}^{n-2}C_2 + {}^{n-2}C_4 + {}^{n-2}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\} + x_3\{{}^{n-3}C_2 + {}^{n-3}C_4 + {}^{n-3}C_6 \ldots\ldots\ldots\ldots\} + \ldots\ldots\ldots\ldots\ldots x_{n-2}\{{}^2C_2\} + x_1 + x_2 \ldots\ldots\ldots\ldots\ldots x_{n-1} + x_n$

the above relation can be simplified as

$q = x_1\{1 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\} + x_2\{1 + {}^{n-2}C_2 + {}^{n-2}C_4 + {}^{n-2}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\} + x_3\ \{1 + {}^{n-3}C_2 + {}^{n-3}C_4 + {}^{n-3}C_6 \ldots\ldots\ldots\ldots\} \ldots\ldots\ldots\ldots. + x_{n-2}\{1 + {}^2C_2\} + x_{n-1} + x_n$

the above relation can be further rewritten as

$q = x_1\{{}^{n-1}C_0 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\} + x_2\{{}^{n-2}C_0 + {}^{n-2}C_2 + {}^{n-2}C_4 + {}^{n-2}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\} + x_3\ \{{}^{n-3}C_0 + {}^{n-3}C_2 + {}^{n-3}C_4 + {}^{n-3}C_6 \ldots\ldots\ldots\ldots\} + \ldots\ldots\ldots\ldots x_{n-2}\{{}^2C_0 + {}^2C_2\} + x_{n-1} + x_n$

Now let us take <u>Denominator</u>

i .e

The product of GCDs of all sets containing even no. of Elements × LCM(of all numbers)

Let

ISSN 2229-5518

The product of GCDs of all sets containing even no. of Elements × LCM(of all numbers) = 2^u

So we take elements as A= {$2^\wedge(x_1)$,$2^\wedge(x_2)$…$2^\wedge(x_{n-1})$,$2^\wedge(x_n)$}

$(x_1 \le x_2 \le x_3 \ldots\ldots x_{n-1} \le x_n)$

Let solution is =2^u

So no. of sets containing even no. of elements with $2^\wedge(x_1)$ as GCD are :

With 2no`s {$2^\wedge(x_1)$, a } a is any other number from set A.

There can be ${}^{n-1}C_1$ such sets possible

With 4no`s {$2^\wedge(x_1)$, a, b, c,} a, b and c are any 3 numbers from set A. There can be ${}^{n-1}C_3$ such sets possible

……………and so on.

So no of sets containing even no. of elements with $2^\wedge(x_2)$ as GCD are:

With 2no`s {$2^\wedge(x_2)$, a, } a is any number from set A.

There can be ${}^{n-2}C_1$ such sets possible

With 4no`s {$2^\wedge(x_2)$, a, b, c,} a, b and c are any 3 numbers from seta. There can be ${}^{n-2}C_3$ such sets possible

………………and so on

Solution =2^u

If we multiply GCD s(even) and LCM of the numbers, we get

$u = x_1\{{}^{n-1}C_1 + {}^{n-1}C_3 + {}^{n-1}C_5 + {}^{n-1}C_7 \ldots\ldots\} + x_2\{{}^{n-2}C_1 + {}^{n-2}C_3 + {}^{n-2}C_5 + {}^{n-2}C_7 \ldots\ldots\} + x_3\{{}^{n-3}C_1 + {}^{n-3}C_3 + {}^{n-3}C_5 + {}^{n-3}C_7 \ldots\ldots\} + \ldots\ldots x_{n-1}\{{}^{1}C_1\} + x_n$ (x_n is LCM of all numbers)

If this relation is correct,then

Numerator/Denominator = 1

Substituting values, we get

2^q / 2^u = 1

2^(q–u) = 1

q–u = 0

So if q–u = 0 is proved,the above relation is true for n numbers

Let us check this

Substituting values of q and u

We get

$[x_1\{{}^{n-1}C_0+{}^{n-1}C_2+{}^{n-1}C_4+{}^{n-1}C_6 \ldots\ldots\} + x_2\{{}^{n-2}C_0+{}^{n-2}C_2+{}^{n-2}C_4+{}^{n-2}C_6 \ldots\} + x_3\{{}^{n-3}C_0+{}^{n-3}C_2+{}^{n-3}C_4+{}^{n-3}C_6 \ldots\ldots\} + \ldots\ldots x_{n-2}\{{}^{2}C_0+{}^{2}C_2\} + x_{n-1} + x_n] - [x_1\{{}^{n-1}C_1 + {}^{n-1}C_3 + {}^{n-1}C_5 + {}^{n-1}C_7 + \ldots\ldots\} + x_2\{{}^{n-2}C_1+{}^{n-2}C_3+{}^{n-2}C_5+{}^{n-2}C_7 \ldots\} + x_3\{{}^{n-3}C_1+{}^{n-3}C_3+{}^{n-3}C_5+{}^{n-3}C_7 \ldots\ldots\} + \ldots\ldots x_{n-1}\{{}^{1}C_1\} + x_n]$

$= [x_1\{{}^{n-1}C_0 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\ldots\} - \{ {}^{n-1}C_1 + {}^{n-1}C_3 + {}^{n-1}C_5 + {}^{n-1}C_7 \ldots\ldots\}] + [x_2\{{}^{n-2}C_0 + {}^{n-2}C_2 + {}^{n-2}C_4 + {}^{n-2}C_6 \ldots\} - \{{}^{n-2}C_1+{}^{n-2}C_3+{}^{n-2}C_5+{}^{n-2}C_7 \ldots\ldots\}] \ldots + x_n - x_n + x_{n-1} - x_{n-1}$

since we know the relation

"$({}^{n}C_0+{}^{n}C_2+{}^{n}C_4+{}^{n}C_6 \ldots\ldots) = ({}^{n}C_1+{}^{n}C_3+{}^{n}C_5+{}^{n}C_7 \ldots\ldots)$"

➔ $({}^{n}C_0+{}^{n}C_2+{}^{n}C_4+{}^{n}C_6 \ldots\ldots) - ({}^{n}C_1+{}^{n}C_3+{}^{n}C_5+{}^{n}C_7 \ldots\ldots) = 0$

using above relation

$[x_1\{{}^{n-1}C_0 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\ldots\} - \{ {}^{n-1}C_1 + {}^{n-1}C_3 + {}^{n-1}C_5 + {}^{n-1}C_7 \ldots\ldots\}] + [x_2\{{}^{n-2}C_0+{}^{n-2}C_2+ {}^{n-2}C_4 + {}^{n-2}C_6 \ldots\} - \{{}^{n-2}C_1 + {}^{n-2}C_3 + {}^{n-2}C_5 + {}^{n-2}C_7 \ldots\}] \ldots\ldots + x_n - x_n + x_{n-1} - x_{n-1}$

i.e.,

$(x_1 \times 0)+(x_2 \times 0) \ldots\ldots +0 =0$

So q-u =0 ,

Similarly for all other primes(3,5,7….etc) and their powers it holds true.

Similarly for all numbers because every number is product of primes and their powers.

$$1 = \frac{a_1 \times a_2 \times a_3 \ldots\ldots a_{n-1} \times a_n \times \text{The product of GCDs of all sets containing odd no. of elements(except 1)}}{\text{The product of GCDs of all sets containing even no. of Elements} \times \text{LCM(all numbers)}}$$

i.e.,

$a_1 \times a_2 \times a_3 \ldots\ldots a_{n-1} \times a_n$ =

$$\frac{\text{The product of GCDs of all sets containing even no. of Elements} \times \text{LCM(all numbers.)}}{\text{The product of GCDs of all sets containing odd no. of elements (except 1)}}$$

Hence Relation 1 is proved

4 Proof of Relation 2

2nd Relation

Now let us prove the 2nd relation i.e.

$a_1 \times a_2 \times a_3 \ldots\ldots a_{n-1} \times a_n$ =

The product of LCMs of all sets containing even no. of Elements × GCD(all numbers.)

The product of LCMs of all sets containing odd no. of elements (except 1)

Let $a_1,a_2,a_3\ldots\ldots\ldots a_{n-1},a_n$ be 'n' natural numbers

The above equation can be rearranged as

$a_1\times a_2\times a_3\ldots\ldots\ldots a_{n-1}$ x a_n × The product of LCMs of all sets containing odd no. of elements(except 1)

1= ———————————————————————

The product of LCMs of all sets containing even no. of Elements × GCD(of all numbers)

Let

$a_1 = 2^{(x_1)}\times 3^{(y_2)}\times 5^{(z_3)}\ldots\ldots\ldots\ldots\ldots$

$a_2 = 2^{(x_3)}\times 3^{(y_4)}\times 5^{(z_1)}\ldots\ldots\ldots\ldots\ldots$

:

:

:

$a_n = 2^{(x_n)}\times 3(y_1)\times 5^{(z_{n-1})}\times\ldots\ldots\ldots\ldots\ldots$

(all numbers are taken in random)

(all exponents are whole numbers only)

Let us assume

$x_1 \geq x_2 \geq x_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots x_{n-1} \geq x_n$

$y_1 \geq y_2 \geq y_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots y_{n-1} \geq y_n$

and so on.

Similarly with all other exponents of prime factors.

Let us take <u>Numerator</u>

$a_1\times a_2\times a_3\ldots\ldots\ldots a_{n-1}\times a_n$ × The product of GCDs of all sets

containing odd no. of elements(except 1) = $2^q\times 3^r\times 5^s\ldots p^t$

For some $\{q, r, s\ldots\ldots\ldots\ldots\ldots\ldots\ldots t\} \in N$

LCM (or GCD) of n numbers = Product of LCM (or GCD) of each of its prime factors with same base.

So for 2 powers we apply the above relation

We take elements as A= $\{2^{(x_1)},2^{(x_2)}\ldots\ldots 2^{(x_{n-1})},2^{(x_n)}\}$

$(x_1 \geq x_2 \geq x_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots x_{n-1} \geq x_n)$

Let solution is $=2^q$

To know the value of q it is necessary to find the number of sets

No. of sets containing odd no. of elements with $2^{(x_1)}$ as LCM are :

With 3no`s $\{2^{(x_1)}, a, b\}$ a and b are any two numbers of set A. There can be ${}^{n-1}C_2$ such sets possible

With 5no`s $\{2^{(x_1)}, a, b, c, d\}$ a, b, c and d are any 4 numbers of set A. There can be ${}^{n-1}C_4$ such sets possible

………… and so on.

No of sets containing odd no. of elements with $2^{(x_2)}$ as LCM are:

With 3no`s $\{2^{(x_2)}, a, b\}$ a and b are any two numbers of set A. There can be ${}^{n-2}C_2$ such sets possible

With 5no`s $\{2^{(x_2)}, a, b, c, d\}$ a, b, c and d are any 4 numbers set A. There can be ${}^{n-2}C_4$ such sets possible

………………..and so on

Solution $=2^q$

If we multiply LCMs (odd) and product of the numbers, we get

$q = x_1\{{}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\} + x_2\{{}^{n-2}C_2 + {}^{n-2}C_4 + {}^{n-2}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\} + x_3 \{{}^{n-3}C_2 + {}^{n-3}C_4 + {}^{n-3}C_6\ldots\ldots\ldots\ldots\} + \ldots\ldots\ldots\ldots x_{n-2}\{{}^2C_2\} + x_1 + x_2 \ldots\ldots\ldots\ldots\ldots x_{n-1} + x_n$

the above relation can be simplified as

$q = x_1\{1 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\} + x_2\{1 + {}^{n-2}C_2 + {}^{n-2}C_4 + {}^{n-2}C_6\ldots\ldots\ldots\ldots\ldots\ldots\} + x_3 \{1+ {}^{n-3}C_2 + {}^{n-3}C_4 + {}^{n-3}C_6 \ldots\ldots\ldots\ldots\} \ldots\ldots\ldots\ldots + x_{n-2}\{1+{}^2C_2\} + x_{n-1} + x_n$

the above relation can be further rewritten as

$q = x_1\{{}^{n-1}C_0 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\} + x_2\{{}^{n-2}C_0 + {}^{n-2}C_2 + {}^{n-2}C_4 + {}^{n-2}C_6\ldots\ldots\ldots\ldots\ldots\ldots\} + x_3 \{{}^{n-3}C_0 + {}^{n-3}C_2 + {}^{n-3}C_4 + {}^{n-3}C_6\ldots\ldots\ldots\ldots\} + \ldots\ldots\ldots\ldots x_{n-2}\{{}^2C_0 + {}^2C_2\} + x_{n-1} + x_n$

Now let us discuss about Denominator

i .e

The product of LCMs of all sets containing even no. of Elements × GCD (of all numbers)

Let

The product of LCMs of all sets containing even no. of Elements × GCD(of all numbers) = 2^u

So we take elements as A= $\{2^{(x_1)},2^{(x_2)}\ldots 2^{(x_{n-1})},2^{(x_n)}\}$

$(x_1 \geq x_2 \geq x_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots x_{n-1} \geq x_n)$

Let solution is $=2^u$

So no. of sets containing even no. of elements with $2^{(x_1)}$ as LCM are

With 2no`s $\{2^{(x_1)}, a\}$ a is any other number from set A.

There can be ${}^{n-1}C_1$ such sets possible

With 4no`s $\{2^{(x_1)}, a, b, c,\}$ a, b and c are any 3 numbers from set A. There can be ${}^{n-1}C_3$ such sets possible

…………….and so on.

So no of sets containing even no. of elements with $2^{(x_2)}$ as LCM are

With 2no`s $\{2^{(x_2)}, a,\}$ a is any number from set A.

There can be ${}^{n-2}C_1$ such sets possible

With 4no`s $\{2^{(x_2)}, a, b, c,\}$ a, b and c are any 3 numbers from set A. There can be ${}^{n-2}C_3$ such sets possible

……………….and so on

Solution $=2^u$

If we multiply LCM s(even) and GCD of the numbers, we get

$u = x_1\{{}^{n-1}C_1 + {}^{n-1}C_3 + {}^{n-1}C_5 + {}^{n-1}C_7 \ldots\} + x_2\{{}^{n-2}C_1 + {}^{n-2}C_3 + {}^{n-2}C_5 + {}^{n-2}C_7 \ldots\} + x_3\{{}^{n-3}C_1 + {}^{n-3}C_3 + {}^{n-3}C_5 + {}^{n-3}C_7 \ldots\} + \ldots x_{n-1}\{{}^1C_1\} + x_n$ (x_n is GCD of all numbers)

If this relation is correct,then

Numerator/Denominator = 1

Substituting values, we get

$2^q / 2^u = 1$

$2^{(q-u)} = 1$

$q-u = 0$

So if $q-u = 0$ is proved,the above relation is true for n numbers

Let us check this

Substituting values of q and u

We get

$[x_1\{{}^{n-1}C_0+{}^{n-1}C_2+{}^{n-1}C_4+{}^{n-1}C_6 \ldots\} + x_2\{{}^{n-2}C_0+{}^{n-2}C_2+{}^{n-2}C_4+{}^{n-2}C_6 \ldots\} + x_3\{{}^{n-3}C_0+{}^{n-3}C_2+{}^{n-3}C_4+{}^{n-3}C_6 \ldots\} + \ldots x_{n-2}\{{}^2C_0+{}^2C_2\} + x_{n-1} + x_n] - [x_1\{{}^{n-1}C_1 + {}^{n-1}C_3 + {}^{n-1}C_5 + {}^{n-1}C_7 + \ldots\} + x_2\{{}^{n-2}C_1+{}^{n-2}C_3+{}^{n-2}C_5+{}^{n-2}C_7 \ldots\} + x_3\{{}^{n-3}C_1+{}^{n-3}C_3+{}^{n-3}C_5+{}^{n-3}C_7 \ldots\} + \ldots x_{n-1}\{{}^1C_1\} + x_n]$

$= [x_1\{{}^{n-1}C_0 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\} - \{{}^{n-1}C_1 + {}^{n-1}C_3 + {}^{n-1}C_5 + {}^{n-1}C_7 \ldots\}] + [x_2\{{}^{n-2}C_0 + {}^{n-2}C_2 + {}^{n-2}C_4 + {}^{n-2}C_6 \ldots\} - \{{}^{n-2}C_1+{}^{n-2}C_3+{}^{n-2}C_5+{}^{n-2}C_7 \ldots\}] \ldots + x_n - x_n + x_{n-1} - x_{n-1}$

since we know the relation

"$({}^nC_0+{}^nC_2+{}^nC_4+{}^nC_6 \ldots) = ({}^nC_1+{}^nC_3+{}^nC_5+{}^nC_7 \ldots)$"

$\Rightarrow ({}^nC_0+{}^nC_2+{}^nC_4+{}^nC_6 \ldots) - ({}^nC_1+{}^nC_3+{}^nC_5+{}^nC_7 \ldots) = 0$

using above relation

$[x_1\{{}^{n-1}C_0 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \ldots\} - \{{}^{n-1}C_1 + {}^{n-1}C_3 + {}^{n-1}C_5 + {}^{n-1}C_7 \ldots\}] + [x_2\{{}^{n-2}C_0+{}^{n-2}C_2+{}^{n-2}C_4 + {}^{n-2}C_6 \ldots\} - \{{}^{n-2}C_1 + {}^{n-2}C_3 + {}^{n-2}C_5 + {}^{n-2}C_7 \ldots\}] \ldots + x_n - x_n + x_{n-1} - x_{n-1}$

i.e.,

$(x_1 \times 0)+(x_2 \times 0) \ldots +0 = 0$

So $q-u = 0$,

Similarly for all other primes(3,5,7….etc) and their powers it holds true.

Similarly for all numbers because every number is product of primes and their powers.

$$1 = \frac{a_1 \times a_2 \times a_3 \ldots a_{n-1} \times a_n \times \text{The product of LCMs of all sets containing odd no. of elements(except 1)}}{\text{The product of LCMs of all sets containing even no. of Elements} \times \text{GCD(all numbers)}}$$

i.e.,

$$a_1 \times a_2 \times a_3 \ldots a_{n-1} \times a_n = \frac{\text{The product of LCMs of all sets containing even no. of Elements} \times \text{GCD(of all numbers.)}}{\text{The product of LCMs of all sets containing odd no. of elements (except 1)}}$$

Hence Relation 2 is proved

Example

Let us take one example with Five numbers (n=5) i.e,
$a_1=24$, $a_2=36$, $a_3=45$, $a_4=85$, $a_5=135$

Product of a_1 , a_2 , a_3 , a_4 , a_5 = 446148000

Two nos.taken	GCD	LCM
24,36	12	72
24,45	3	360
24,85	1	2040
24,135	3	1080
36,45	9	180
36,85	1	3060
36,135	9	540
45,85	5	765
45,135	45	135

85,135	5	2295
Product(Two)	**9841500**	**4.02582E+27**

Three Nos.	**GCD**	**LCM**
24,36,45	3	360
24,36,85	1	6120
24,36,135	3	1080
24,45,85	1	6120
24,45,135	3	1080
24,85,135	1	18360
36,45,85	1	3060
36,45,135	9	540
36,85,135	1	9180
45,85,135	5	2295
Product(Three)	**1215**	**1.00523E+34**

Four Nos.	**GCD**	**LCM**
24,36,45,85	1	6120
24,36,45,135	3	1080
24,36,85,135	1	18360
24,45,85,135	1	18360
36,45,85,135	1	9180
Product(Four)	**3**	**2.04533E+19**

Five Nos.	**GCD**	**LCM**
24,36,45,85,135	1	18360
Product(Five)	**1**	**18360**

Relation 1 = (9841500 * 3 * 18360) / (1215 * 1) = **446148000**

Relation 2 = (4.02582E+27 * 2.04533E+19 * 1) / 1.00523E+34 * 18360) = **446148000**

Conclusion

Both relations are tested for different values from n = 4 to 10 in MS-Excel sheet and found to be correct.

Acknowledgements

The author wishes to thank Mr. H.Srinivas Reddy,M.Sc,B.Ed (Varadhi Institute of Mathematics & Science, Karimnagar,A.P.) for his guidance and support. The author also wishes to thank Rev.Fr.Casmir D'Souza, Head Master of St.Claire High School, Ramagundam, Dist:Karimnagar, A.P.(India) and Dr. S.R.Santhanam (AMTI, Chennai, India) for their co-operation and encouragement.