Relation between N numbers and their LCMs and GCDs

C.S.Abhishek

Abstract— This paper explains the relation among product of 'n' natural numbers and the LCMs and GCDs of all combinations of these numbers. It is known that the product of two numbers is equal to the product of their LCM and GCD. There is also a relation for three numbers. In this paper I am going to introduce two generalized relations for 'n' natural numbers.

Index Terms — N numbers , Product , LCM (Least Common Multiple) , GCD (Greatest Common Multiple) , Relation-1, Relation-2 , Odd , Even

1 Introduction

2 Relations

We know that if a, b are two numbers then their product is equal to product of their LCM and GCD

i.e. $axb = LCM(a,b) \times GCD(a,b)$

and also we know that if a,b,c are 3 numbers then

LCM $(a,b,c) = [a \times b \times c \times GCD (a,b,c)] / [GCD (a,b) \times GCD (b,c) \times GCD(c,a)]$

GCD $(a,b,c) = [a \times b \times c \times LCM (a,b,c)] / [LCM (a,b) \times LCM (b,c) \times LCM(c,a)]$

In this paper I am going to introduce two generalized formulae and their proof for 'n' natural numbers. The product of 'n' numbers can be expressed in terms of LCMs and GCDs of different combinations in two different ways as Relation 1 and Relation 2. The Product of 'n' numbers can be expressed in two forms i.e, Relation 1 and Relation 2 as below:

Let $a_{1,}a_{2,}a_{3,}\dots a_{n-1}$, a_{n} be 'n' natural numbers

Relation - 1

 $a_1 \times a_2 \times a_3 \dots a_{n-1} \times a_n =$

The product of GCDs of all sets containing

even no. of Elements × LCM(of all numbers.)

The product of GCDs of all sets containing odd no. of elements (except 1)

Relation - 2

 $a_1 \times a_2 \times a_3 \dots a_{n-1} \times a_n =$

The product of LCMs of all sets containing

even no. of Elements × GCD(of all numbers.)

The product of LCMs of all sets containing odd no. of elements (except 1)

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3 Proof of Relation 1

1st Relation



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Now let us prove the 1st Relation i.e.

 $a_1 \times a_2 \times a_3 \dots a_{n-1} \times a_n =$

The product of GCDs of all sets containing

even no. of Elements × LCM(of all numbers.)

The product of GCDs of all sets containing odd no. of elements (except 1)

Let $a_1, a_2, a_3, \dots, a_{n-1}, a_n$ be 'n' natural numbers

The above equation can be rearranged as

 $a_1 \times a_2 \times a_3$ $a_{n-1} \times a_n \times$ The product of GCDs of all sets containing odd no. of elements(except 1)

1=

The product of GCDs of all sets containing even no. of Elements × LCM(of all numbers)

Let

 $a_1 = 2^{(x_1) \times 3^{(y_2) \times 5^{(z_3)}}....a_2 = 2^{(x_3) \times 3^{(y_4) \times 5^{(z_1)}}....a_2$

 $a_n = 2^{(x_n) \times 3(y_1) \times 5^{(z_{n-1}) \times \dots + y_{n-1} \times \dots$

(all numbers are taken in random)

(all exponents are whole numbers only)

Let us assume

 $x_1 \le x_2 \le x_3$ $x_{n-1} \le x_n$

 $y_1 \leq y_2 \leq y_3.\dots..y_{n-1} \leq y_n$

and so on.

Similarly with all other exponents of prime factors.

Let us take <u>Numerator</u>

 $a_1 \times a_2 \times a_3 \dots a_{n-1} \times a_n \times$ The product of GCDs of all sets

containing odd no. of elements(except 1) = $2^{q \times 3^{r} \times 5^{s}}$p^t

For some $\{q, r, s, \dots, t\} \in \mathbb{N}$

LCM (or GCD) of n numbers = Product of LCM (or GCD) of each of its prime factors with same base.

So for 2 powers we apply the above relation

We take elements as A= $\{2^{(x_1)}, 2^{(x_2)}, \dots, 2^{(x_{n-1})}, 2^{(x_n)}\}$

 $(x_1 \le x_2 \le x_3, \dots, x_{n-1} \le x_n)$

Let solution is =2q

To know the value of q it is necessary to find the number of sets

No. of sets containing odd no. of elements with $2^{(x_1)}$ as GCD are :

With 3no's $\{2^{(x_1)}, a, b\}$ a and b are any two numbers of set A. There can be $n^{-1}C_2$ such sets possible.

With 5no`s $\{2^{(x_1)}, a, b, c, d\} a, b, c and d are any 4$ numbers of set A. There can be $n^{-1}C_4$ such sets possible

..... and so on.

No of sets containing odd no. of elements with $2^{(x_2)}$ as GCD are :

With 3no's $\{2^{(x_2)}, a, b\}$ a and b are any two numbers of set A. There can be $n^{-2}C_2$ such sets possible

 $\begin{array}{ll} \mbox{With 5no`s} & \{2^{(x_2)}, a, b, c, d\} \ a, b, c \ and \ d \ are \ any \ 4 \\ & \ numbers \ set \ A. \ There \ can \ be \ {}^{n-2}C_4 \ \ such \ sets \ possible \end{array}$

.....and so on

Solution =2q

If we multiply GCD s (odd) and product of the numbers, we get

$q = x_1 \{ n^{-1}C_2 + n^{-1}C_4 + n^{-1}C_6 \dots \dots \dots \}$	
$^{n-2}C_4$ + $^{n-2}C_6$	
$+n^{-3}C_4+n^{-3}C_6$	C_2 + x_1 + x_2
$x_{n-1} + x_n$	

the above relation can be simplified as

$q = x_1 \{1 + n^{-1}C_2 + n^{-1}C_4 + n^{-1}C_6 \dots \dots$	} +
$x_2\{1+n-2C_2+n-2C_4+n-2C_6,\dots,k\}$ +	$x_3 \{1 + n^{-3}C_2 + n^{-3}C_4\}$
$+n^{-3}C_6$	$+ x_{n-1} + x_n$

the above relation can be further rewritten as

q	=	$x_1 \{ {}^{n-1}C_0 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6$	} +
x_2^{r}	$-2C_0$	$+n^{-2}C_2 + n^{-2}C_4 + n^{-2}C_6$	}+ $x_3 \{ n^{-3}C_0 + n^{-3}C_2 + \dots \}$
n-3($C_4 + n^{-1}$	$^{-3}C_6$	$x_{n-2}^{2}C_{0}^{+2}C_{2}^{+} + x_{n-1}^{-} + x_{n}^{-}$

Now let us take <u>Denominator</u>

i.e

The product of GCDs of all sets containing even no. of Elements × LCM(of all numbers)

Let

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The product of GCDs of all sets containing even no. of Elements \times LCM(of all numbers) = 2^{u}

So we take elements as A= $\{2^{(x_1)}, 2^{(x_2)}, \dots, 2^{(x_{n-1})}, 2^{(x_n)}\}$

 $(x_1 \le x_2 \le x_3, \dots, x_{n-1} \le x_n)$

Let solution is $=2^{u}$

So no. of sets containing even no. of elements with $2^{(x_1)}$ as GCD are :

With 2no's $\{2^{(x_1)}, a\}$ a is any other number from set A.

There can be $n^{-1}C_1$ such sets possible

With 4no's ${2^{(x_1)}, a, b, c}$ a, b and c are any 3 numbers from set A. There can be ${}^{n-1}C_3$ such sets possible

.....and so on.

So no of sets containing even no. of elements with $2^{(x_2)}$ as GCD are:

With 2no's $\{2^{(x_2)}, a, \}$ a is any number from set A.

There can be $n^{-2}C_1$ such sets possible

With 4no's $\{2^{(x_2)}, a, b, c, \}a$, b and c are any 3 numbers from seta. There can be ${}^{n-2}C_3$ such sets

possible

.....and so on

Solution = 2^u

If we multiply GCD s(even) and LCM of the numbers, we get

 $\begin{array}{rcl} u &=& x_1 \{ {}^{n-1}C_1 &+& {}^{n-1}C_3 &+& {}^{n-1}C_5 &+& {}^{n-1}C_7 &\dots \\ &+& x_2 \{ {}^{n-2}C_1 + {}^{n-2}C_3 + {}^{n-2}C_5 + {}^{n-2}C_7 &\dots \\ &+& x_3 \{ {}^{n-3}C_1 + {}^{n-3}C_3 &+& {}^{n-3}C_5 &+& {}^{n-3}C_7 &\dots \\ &+& x_{n-1} \{ {}^{1}C_1 \} + x_n & (x_n \text{ is LCM of all numbers }) \end{array}$

If this relation is correct, then

Numerator/Denominator = 1

Substituting values, we get

 $2^q / 2^u = 1$

 $2^{(q-u)} = 1$

q - u = 0

So if q-u = 0 is proved, the above relation is true for n numbers

Let us check this

Substituting values of q and u

We get

 $\begin{array}{l} [x_1\{^{n-1}C_0+^{n-1}C_2+^{n-1}C_4+^{n-1}C_6, \ldots, + x_3\{^{n-3}C_0+^{n-3}C_2+^{n-3}C_4+^{n-3}C_6 \\ \ldots, + x_2\{^{n-2}C_0+^{n-2}C_2+^{n-2}C_4+^{n-2}C_6 \\ \ldots, + x_{n-2}\{^2C_0+^2C_2\} + x_{n-1} + x_n] \\ + x_{n-1}\{x_1\{^{n-1}C_1+^{n-1}C_3+^{n-1}C_5+^{n-1}C_7+, \ldots, + x_3\{^{n-3}C_1+^{n-3}C_3+^{n-3}C_5+^{n-3}C_7, \\ \ldots, + x_{n-1}\{^{n-2}C_1+^{n-2}C_3+^{n-2}C_5+^{n-2}C_7, \\ \ldots, + x_{n-1}\{^{n-3}C_1+^{n-3}C_3+^{n-3}C_5+^{n-3}C_7, \\ \ldots, + x_{n-1}\{^{n-3}C_1+^{n-3}C_3+^{n-3}C_7+^{n-$

$= [x_1 \{ n^{-1}C_0 + n^{-1}C_2 + n^{-1}C_4 + n^{-1}C_6 + n^{-1}C$	
$+ {}^{n-1}C_5 + {}^{n-1}C_7 \dots$	}] + $[x_2\{n^{-2}C_0 + n^{-2}C_2 + n^{-2}C_4]$
+ $n^{-2}C_6$ } - { $n^{-2}C_1$ + $n^{-2}C_3$ + n^{-1}	$^{2}C_{5} + ^{n-2}C_{7} \dots $
$\dots + x_n - x_n + x_{n-1} - x_{n-1}$	

since we know the relation

$$"({}^{n}C_{0}+{}^{n}C_{2}+{}^{n}C_{4}+{}^{n}C_{6}.....)=({}^{n}C_{1}+{}^{n}C_{3}+{}^{n}C_{5}+{}^{n}C_{7}......)"$$

$$({}^{n}C_{0} + {}^{n}C_{2} + {}^{n}C_{4} + {}^{n}C_{6} \dots) - ({}^{n}C_{1} + {}^{n}C_{3} + {}^{n}C_{5} + {}^{n}C_{7} \dots) = 0$$

using above relation

$$\begin{array}{l} \left[x_1 \{^{n-1}C_0 + \ ^{n-1}C_2 + \ ^{n-1}C_4 + \ ^{n-1}C_6 & \ldots & \ldots \end{array} \right\} - \left\{ \begin{array}{l} n^{-1}C_1 + \ ^{n-1}C_3 + \ ^{n-1}C_5 + \ ^{n-1}C_7 & \ldots & \ldots \end{array} \right\} \\ \left[x_2 \{^{n-2}C_0 + \ ^{n-2}C_2 + \ ^{n-2}C_4 + \ ^{n-2}C_6 & \ldots & \ldots \right\} - \left\{ \begin{array}{l} n^{-2}C_1 + \ ^{n-2}C_3 + \ ^{n-2}C_5 + \ ^{n-2}C_7 & \ldots & \ldots \end{array} \right\} \\ \left. \ldots & \ldots & x_n - x_n + x_{n-1} - x_{n-1} \end{array} \right]$$

i.e.,

 $(x_1 \times 0) + (x_2 \times 0) + (x_2 \times 0) + 0 = 0$

So q-u =0,

Similarly for all other primes(3,5,7....etc) and their powers it holds true.

Similarly for all numbers because every number is product of primes and their powers.

 $a_1 \times a_2 \times a_3$ $a_{n-1} \times a_n \times$ The product of GCDs of all sets containing odd no. of elements(except 1)

The product of GCDs of all sets containing even no. of Elements × LCM(all numbers)

i.e.,

 $a_1 \times a_2 \times a_3 \dots a_{n-1} \times a_n =$

The product of GCDs of all sets containing even no. of Elements × LCM(all numbers.)

The product of GCDs of all sets containing odd no. of

elements (except 1)

Hence Relation 1 is proved

4 Proof of Relation 2

2nd Relation

Now let us prove the 2nd relation i.e.

 $a_1 \times a_2 \times a_3 \dots a_{n-1} \times a_n =$

The product of LCMs of all sets containing even no. of Elements × GCD(all numbers.)

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The product of LCMs of all sets containing	With 3no's $\{2^{(x_1)}, a, b\}$ a and b are any two numbers of set A. There can be $n^{-1}C_2$ such sets possible	
odd no. of elements (except 1)	With 5no's $\{2^{(x_1)}, a, b, c, d\}$ a, b, c and d are any 4 numbers of set A. There can be $n^{-1}C_4$ such	
Let $a_1,a_2,a_3,\ldots,a_{n-1},a_n$ be 'n' natural numbers	sets possible	
The above equation can be rearranged as	and so on.	
$a_1 \times a_2 \times a_3$ $a_{n-1} \times a_n \times$ The product of LCMs of all sets containing odd no. of elements(except 1)	No of sets containing odd no. of elements with $2^{(x_2)}$ as LCM are:	
1= The product of LCMs of all sets containing even	With 3no`s $\{2^{(x_2)}, a, b\}$ a and b are any two numbers of set A. There can be ${}^{n-2}C_2$ such sets possible	
no. of Elements × GCD(of all numbers) Let	With 5no`s $\{2^{(x_2)}, a, b, c, d\} a, b, c and d are any 4 numbers set A. There can be n^{-2}C_4 such sets possible$	
$a_1 = 2^{(x_1) \times 3^{(y_2) \times 5^{(z_3)}}$	and so on	
$a_2 = 2^{(x_3)} \times 3^{(y_4)} \times 5^{(z_1)}$		
:	Solution =2 ^q	
:	If we multiply LCMs (odd) and product of the numbers, we get	
$a_n=2^{(x_n)\times 3(y_1)\times 5^{(z_{n-1})\times \dots}}$	$\begin{array}{l} q = x_1 \{ {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \dots \} + x_2 \{ {}^{n-2}C_2 + {}^{n-2}C_4 \\ {}^{n-2}C_6 \dots \} + x_3 \{ {}^{n-3}C_2 + {}^{n-3}C_4 + {}^{n-3}C_6 \dots \\ + \dots \dots x_{n-2} \{ {}^2C_2 \} + x_1 + x_2 \dots x_{n-1} + x_n \end{array}$	
(all numbers are taken in random)	the above relation can be simplified as	
(all exponents are whole numbers only)	$q = x_1 \{ 1 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \dots \} + x_2 \{ 1 + {}^{n-2}C_2 \}$	
Let us assume	+ $n^{-2}C_4$ + $n^{-2}C_6$ } + $x_3 \{1 + n^{-3}C_2 + n^{-3}C_4 + n^{-3}C_6$ }	
$x_1 \ge x_2 \ge x_3 \dots \dots x_{n-1} \ge x_n$		
$y_1 \ge y_2 \ge y_3 y_{n-1} \ge y_n$	the above relation can be further rewritten as	
and so on.		
Similarly with all other exponents of prime factors.	$q = x_1 \{ {}^{n-1}C_0 + {}^{n-1}C_2 + {}^{n-1}C_4 + {}^{n-1}C_6 \dots \} +$	
Let us take <u>Numerator</u>	$x_2 \{ n^{-2}C_0 + n^{-2}C_2 + n^{-2}C_4 + n^{-2}C_6 \dots \} + x_3 \{ n^{-3}C_0 + n^{-3}C_0 + n^{-3}C_0 \}$	
$a_1 \times a_2 \times a_3$ $a_{n-1} \times a_n \times$ The product of GCDs of all sets	$ \begin{array}{l} n^{-3}C_{2} + n^{-3}C_{4} + n^{-3}C_{6} \dots \\ + x_{n} \end{array} + \dots \\ \begin{array}{l} x_{n-2} \left\{ {}^{2}C_{0} + {}^{2}C_{2} \right\} + x_{n-1} \\ + x_{n} \end{array} $	
containing odd no. of elements(except 1) = $2^{q} \times 3^{r} \times 5^{s} \dots p^{t}$		
For some $\{q, r, s, t\} \in \mathbb{N}$	Now let us discuss about Denominator	
LCM (or GCD) of n numbers = Product of LCM (or GCD) of each of its prime factors with same base.	i.e	
So for 2 powers we apply the above relation	The product of LCMs of all sets containing even no. of Ele- ments × GCD (of all numbers)	
We take elements as A= $\{2^{(x_1)}, 2^{(x_2)}, \dots, 2^{(x_{n-1})}, 2^{(x_n)}\}$	Let	
$(x_1 \ge x_2 \ge x_3x_{n-1} \ge x_n)$	The product of LCMs of all sets containing even no. of Ele-	
Let solution is =2 ^q	ments × GCD(of all numbers) = 2^{u}	
To know the value of q it is necessary to find the number of acta	So we take elements as A= $\{2^{(x_1)}, 2^{(x_2)},, 2^{(x_{n-1})}, 2^{(x_n)}\}$	
sets No. of sets containing odd no. of elements with $2^{(x_1)}$ as LCM	$(x_{1\geq x_{2\geq x_3},\ldots,x_{n-1\geq x_n})$ Let solution is =2 ^u	
are :		

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So no. of sets containing even no. of eler	ments with $2^{(x_1)}$ as	$"({}^{n}C_{0}+{}^{n}C_{2}+{}^{n}C_{4}+{}^{n}C_{6}$)= $({}^{n}C_{1} + {}^{n}C_{3} + {}^{n}C_{3}$	$C_5 + {}^nC_7)''$
LCM are		$lag{(}^{n}C_{0}+^{n}C_{2}+^{n}C_{4$	C_{6})-(${}^{n}C_{1}$ + ${}^{n}C_{3}$	$+^{n}C_{5}+^{n}C_{7}$)=0
With 2no's $\{2^{(x_1)}, a\}$ a is any other num	ber from set A.			
There can be $n^{-1}C_1$ such a	sets possible	using above relation	on	
With 4no's ${2^{(x_1)}, a, b, c}$ a, b and c from set A. There can possible		$ [x_1\{^{n-1}C_0 + n^{-1}C_2 + n^{-1}C_3 + n^{-1}C_5 $	$^{-1}C_4 + {}^{n-1}C_6 \dots$	$ = \{ n^{-1}C_1 + \dots \} \} + [x_2\{n^{-2}C_0 + n^{-2}C_2 + n^{-2}C_5 + n^{-2}C_7, \dots, \}] $
and :	50 on.	$x_{2}C_{4} + x_{2}C_{6}$, .	$+ \frac{1}{2}C_{5} + \frac{1}{2}C_{7}$
So no of sets containing even no. of eler	nents with $2^{(x_2)}$ as	i.e.,		
LCM are	(12)	$(x_1 \times 0) + (x_2 \times 0) \dots$	+0 =0	
With 2no's $\{2^{(x_2)}, a, \}$ a is any number f	from set A.	So $q-u = 0$,		
There can be $n^{-2}C_1$ such	sets possible	-	that $primes(3.5.7)$	etc) and their powers it
With 4no's $\{2^{(x_2)}, a, b, c, \} a, b and c are$	5	holds true.	ther primes(0,0,7	ec) and then powers it
from set A. There ca possible	n be ⁿ⁻² C ₃ such sets	Similarly for all nuprimes and their p		ery number is product of
an Solution =2 ^u	d so on		$\dots a_{n-1} \times a_n \times The$ ining odd no. of ele	e product of LCMs of all
	1		ning oud no. of ex	
If we multiply LCM s(even) and GCD of t	0	1=		
$\begin{array}{rcl} u &=& x_1 \{ {}^{n-1}C_1 &+& {}^{n-1}C_3 &+& {}^{n-1}C_5 &+& {}^{n-1}C_7 & . \\ &+& x_2 \{ {}^{n-2}C_1 &+& {}^{n-2}C_3 &+& {}^{n-2}C_5 &+& {}^{n-2}C_7 . & & \\ &n^{-3}C_3 &+& {}^{n-3}C_5 &+& {}^{n-3}C_7 . & & \\ \end{array}$	}+ $x_3 \{ n^{-3}C_1 + $		t of LCMs of all set no. of Elements ×	s containing even GCD(all numbers)
$x_{n-1}\{{}^{1}C_{1}\} + x_{n}$ (x _n is GCD of all		i.e.,		
If this relation is correct, then	,	$a_1 \times a_2 \times a_3 \dots a_n$	$a_1 \times a_n =$	
Numerator/Denominate	or = 1	The product of LCMs of all sets containing even no. of Elements × GCD(of all numbers.)		
Substituting values, we get				
$2q / 2^u = 1$		The product of LCMs of all sets containing odd no. of elements (except 1)		
$2^{(q-u)} = 1$				
q-u = 0				
So if $q-u = 0$ is proved, the above relation is	s true for n numbers	Hence Relation 2 is	s proved	
Let us check this		Example		
Substituting values of q and u		•		
We get		Let us take one example with Five numbers (n=5) i.e, $a_1=24$, $a_2=36$, $a_3=45$, $a_4=85$, $a_5=135$		mbers (n=5) i.e,
$ [x_1\{^{n-1}C_0 + n^{-1}C_2 + n^{-1}C_4 + n^{-1}C_6 \dots x_2\{^{n-2}C_0 + n^{-2}C_2 + n^{-2}C_4 + n^{-2}C_6 \dots x_2\} + x_3\{^{n-3}C_4 + n^{-2}C_6 \dots x_2\} + x_3\{^{n-3}C_4 + n^{-2}C_6 \dots x_2\} + x_3\{^{n-3}C_4 + n^{-2}C_6 + n^{-2}C_4 + n^{-2}C_6 \dots x_2\} + x_3\{^{n-3}C_4 + n^{-2}C_6 $	$C_0 + n^{-3}C_2 + n^{-3}C_4 + n^{-3}C_6$	Product of a_1, a_2, a_3		00
$x_{n-2} \{ {}^{2}C_{0} \\ [x_{1} \{ {}^{n-1}C_{1} + {}^{n-1}C_{3} + {}^{n-1}C_{5} + {}^{n-1}C_{7} + \\x_{n-2} \} $	$+2C_{2} + x_{n-1} + x_{n} - \frac{1}{2}$	Two nos.taken	GCD	LCM
$x_2\{n^{-2}C_1+n^{-2}C_3+n^{-2}C_5+n^{-2}C_7 \dots + x_3\{n^{-3}C_7+n^{-2}C_7+n^$		24,36	12	72
$x_{21} = c_{11} + c_{31} + c_{51} + c_{7} + \dots + x_{n-1} + x_{n-1}$		24,45	3	360
		24,85	1	2040
		24,135	3	1080
$= [x_1 \{ n^{-1}C_0 + n^{-1}C_2 + n^{-1}C_4 + n^{-1}C_6 \dots \dots \dots]$	\dots - { $n^{-1}C_1 + n^{-1}C_3$	36,45	9	180
$+ n^{-1}C_5 + n^{-1}C_7 \dots$ }] + [x		36,85	1	3060
+ $n^{-2}C_6$		36,135	9	540
+ y - y + y - 1 - y - 1		45.85	5	765

 $\dots + x_n - x_n + x_{n-1} - x_{n-1}$

since we know the relation

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45,85

45,135

5

45

765

135

85,135	5	2295
Product(Two)	9841500	4.02582E+27

Three Nos.	GCD	LCM
24,36,45	3	360
24,36,85	1	6120
24,36,135	3	1080
24,45,85	1	6120
24,45,135	3	1080
24,85,135	1	18360
36,45,85	1	3060
36,45,135	9	540
36,85,135	1	9180
45,85,135	5	2295
Product(Three)	1215	1.00523E+34

Four Nos.	GCD	LCM
24,36,45,85	1	6120
24,36,45,135	3	1080
24,36,85,135	1	18360
24,45,85,135	1	18360
36,45,85,135	1	9180
Product(Four)	3	2.04533E+19

Five Nos.	GCD	LCM
24,36,45,85,135	1	18360
Product(Five)	1	18360

Relation 1 = (9841500 * 3 * 18360) / (1215 * 1) = **446148000**

Relation 2 = (4.02582E+27 * 2.04533E+19 * 1) / 1.00523E+34 * 18360) = **446148000**

Conclusion

Both relations are tested for different values from n = 4 to 10 in MS-Excel sheet and found to be correct.

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